

B Decays - Measurements and Predictions *

Hitoshi Yamamoto

Harvard University

42 Oxford St., Cambridge, MA 02138, USA

Abstract

Hadronic decays of B mesons are reviewed. First, masses of B mesons and observed patterns together with physics behind them are discussed. Then the effective Hamiltonian responsible for major decays is presented and its practical applications are discussed in the context of factorization. Various tests of factorization are then studied. For rare decays, the focus is placed on $K\pi$, $\pi\pi$ final state and the penguin-mediated $X_s\gamma$. In general, the measurements are in excellent agreement with predictions of the standard model.

1 Basic Methods on Upsilon-4S Resonance

Most of the data presented in the following are collected on the upsilon-4S resonance, and some basic experimental techniques are briefly described below.

The B meson pair production cross section on the upsilon-4S resonance is roughly 1 nb; namely, an integrated luminosity of 1 fb^{-1} would generate 1 million B meson pairs. The CLEO-II detector has logged about 1.2 fb^{-1} of data thus generating 1.2 million B meson pairs.

On the upsilon-4S resonance, light quark pairs ($u\bar{u}$, $d\bar{d}$, $s\bar{s}$, and $c\bar{c}$ - often referred to as the ‘continuum’) are also generated in addition to the B meson pairs. The cross section ratio of B meson pair to the continuum is roughly 1 to 2.5. The continuum is often a major background and in order to understand this component, data are taken right below the resonance (32 MeV below the peak) corresponding to about one half of the integrated luminosity taken on the resonance. When we want to plot a distribution of certain parameter for B meson pairs, we can subtract the distribution for the data taken off-resonance from that taken on-resonance (with a proper normalization). The distribution is then said to be ‘continuum subtracted’.

At the upsilon-4S resonance, the B mesons are generated with definite energy and momentum given by

$$E_B = E_{\text{beam}} = 5.289 \text{ GeV}, \quad P_B = 0.325 \text{ GeV}/c \quad (1)$$

*A talk given at ‘B Physics at Hadron Colliders’, Snowmass 1993.

When reconstructing a decay $B \rightarrow f_1 + f_2 + \dots + f_n$, natural parameters to look at are thus the total energy and momentum of the decay products f_i ($i = 1, \dots, n$):

$$E_{\text{tot}} = \sum E_i, \quad P_{\text{tot}} = \sum \vec{P}_i \quad (2)$$

which should peak at E_{beam} and P_B respectively, where E_i and P_i are the energy and momentum of the i -th decay product. In practice, often used parameters are the ‘energy difference’ ΔE and the ‘beam-constrained mass’ M_B defined by

$$\Delta E = E_{\text{tot}} - E_{\text{beam}}, \quad M_B = \sqrt{E_{\text{beam}}^2 - P_{\text{tot}}^2}. \quad (3)$$

Since E_{beam} is a constant, measuring ΔE and M_B is equivalent to measuring E_{tot} and P_{tot} . The mass reconstructed this way has a good resolution which varies from 2.5 to 3.3 MeV depending on decay mode and usually dominated by the spread of beam energy. The essence of this method in background rejection, however, is simply the conservation of energy and absolute momentum in a B meson decay. We will often be referring to M_B and ΔE in the rest of this article; the definitions are as defined above.

2 Masses

2.1 B^- and \bar{B}^0

The masses of neutral and charged B mesons can be measured by fully reconstructing the major decay modes. Figure 1 shows the distribution of the beam-constrained mass M_B for B^- and \bar{B}^0 mesons after requiring that the energy difference ΔE is within 2.5σ of zero. The decay modes used are $B^- \rightarrow D^{*0}\pi^-, D^{*0}\rho^-, D^0\pi^-, D^0\rho^-, \psi K^-$ for the charged B meson and $\bar{B}^0 \rightarrow D^{*+}\pi^-, D^{*+}\rho^-, D^+\pi^-, D^+\rho^-, \psi K^{*0}$ for the neutral B meson. The D^* mesons are detected by the decays $D^{*0} \rightarrow D^0\pi^0$, $D^{*+} \rightarrow D^0\pi^+$ and D mesons are detected by $D^0 \rightarrow K^-\pi^+$, $D^+ \rightarrow K^-\pi^+\pi^+$. These modes are chosen since they are particularly clean. There are 362 signal events for B^- and 340 signal events for \bar{B}^0 . With a correction due to initial state radiation of -1.1 ± 0.5 MeV, we obtain $M_{B^0} = 5280.3 \pm 0.2 \pm 2.0$ MeV and $M_{B^-} = 5279.9 \pm 0.2 \pm 2.0$ MeV. The first error is statistical and the second systematic. The systematic error is dominated by the uncertainty in the energy scale of the storage ring which cancels when we take the mass difference: $M_{B^0} - M_{B^-} = 0.44 \pm 0.25 \pm 0.19$ MeV; namely, the masses of B^- and \bar{B}^0 are consistent with being identical within several tenth of MeV. The results are summarized in Table 1 together with previous measurements.

It is interesting to compare this result with that for strange and charm mesons. There we have $M_{K^0} - M_{K^-} = 4,024 \pm 0.032$ MeV and $M_{D^+} - M_{D^0} = 4.77 \pm 0.27$ MeV [1] which seem to indicate that the meson mass is heavier when a heavy quark is combined with a d quark than with a u quark. The pattern, however, clearly does not repeat for B mesons. The current understanding for the isospin mass splitting is that there are effects due to the $u - d$ mass difference as well as QED effects [5] (i.e. due to the electric charge

(MeV)	CLEO I.5 [2]	ARGUS [3]	CLEO II [4]
M_{B^0}	$5278.0 \pm 0.4 \pm 2.0$	$5279.6 \pm 0.7 \pm 2.0$	$5280.3 \pm 0.2 \pm 2.0$
M_{B^-}	$5278.3 \pm 0.4 \pm 2.0$	$5280.5 \pm 1.0 \pm 2.0$	$5279.9 \pm 0.2 \pm 2.0$
$M_{B^0} - M_{B^-}$	$-0.4 \pm 0.6 \pm 0.5$	$-0.9 \pm 1.2 \pm 0.5$	$0.44 \pm 0.25 \pm 0.19$

Table 1: Masses of neutral and charged B mesons.

Figure 1: The beam-constrained mass for charged (a) and neutral (b) B mesons after ΔE is required to be consistent with zero. Particularly clean modes are selected and summed.

difference between u and d quarks). Both are of order a few MeV, and the two kinds of effects happen to cancel for the B meson case [6]. There seems to be no simple and intrinsic reason to give $M_B^0 = M_B^+$.

2.2 Other Bottom Mesons

Bottom hadrons heavier than B^- and \overline{B}^0 are not produced on upsilon-4S resonance, and the results so far come from accelerators that operate at higher energies.

Figure 2 shows the decay $B_S \rightarrow \psi\phi$, $\phi \rightarrow K^+K^-$ observed by the CDF collaboration [7] in $p\overline{p}$ collisions at 1.8 TeV c.m. energy. There are 14 ± 4.7 events observed and fitting a gaussian to the peak, the B_S mass is determined to be $5383.3 \pm 4.5 \pm 5.0$ MeV. The ALEPH collaboration has also reported a result on B_S mass from two events $B_S \rightarrow \psi'\phi$

Figure 2: $B_S \rightarrow \psi\phi$ decay observed by the CDF collaboration. (a) Invariant mass of $\psi K^+ K^-$ when the $K^+ K^-$ pair forms the ϕ mass (within ± 10 MeV). The dots are for a ϕ mass side band. (b) Invariant mass of $K^+ K^-$ when the $\phi K^+ K^-$ mass is in the B_S peak (within ± 20 MeV).

	Modes	Number of events	M_{B_S} (MeV)
ALEPH [8]	$\psi'\phi, D_S^+\pi^-$	2	$5368.6 \pm 5.6 \pm 1.5$
CDF [7]	$\psi\phi$	14 ± 4.7	$5383 \pm 4.5 \pm 5.0$
OPAL[9]	$\psi\phi$	(1 candidate)	5360 ± 70
DELPHI[10]	$D_S^+(\pi^- \text{ or } a_1^-), \psi\phi$	4	$5357 \pm 12 \pm 6$

Table 2: Measurements of B_S meson mass. The $\psi^{(\prime)}$ and ϕ mesons are detected by $\psi^{(\prime)} \rightarrow l^+l^-$ and $\phi \rightarrow K^+K^-$ respectively, and D_s^+ mesons are detected in the modes $D_S^+ \rightarrow \phi\pi^+, K^*K$.

and $D_S^+\pi^-$. The mass measurement is dominated by the $\phi'\phi$ event and gives $5369 \pm 5.6 \pm 1.5$ MeV. These results are summarized in Table 2 together with a possible candidate event reported earlier by the OPAL collaboration and a recently reported result from DELPHI. The measurements by CDF and ALEPH are marginally consistent (2-sigma difference statistically); taking the weighted average, the mass difference between B_S and B^0 is 97 MeV. The value is strikingly similar to the charm case $M_{D_s^+} - M_{D^+} = 99.5 \pm 0.6$ MeV [1], and also consistent with predictions of non-relativistic models: $M_{B_S} = 5345 - 5388$ MeV [11].

The mass of $B^*(J^P = 1^-)$ has been measured by CUSP [12] and CLEO [13] by detecting the monochromatic photon in the transition $B^* \rightarrow B\gamma$. The numbers are [14]

$$M_{B^*} - M_B = 46.4 \pm 0.3 \pm 0.8 \quad \text{MeV (CLEO)} \quad (4)$$

$$45.6 \pm 1.0 \quad \text{MeV (CUSP)}. \quad (5)$$

These measurements are in accordance with an intriguing observation on the hyperfine splitting

$$\Delta M \equiv M^2(1^-) - M^2(0^-) = \text{const} \approx 0.5 \text{GeV}^2. \quad (6)$$

This holds well for (π, ρ) , K , D , D_S and now for B . In non-relativistic models, such relation is realized when the potential between the constituent quarks is linearly increasing as a function of the distance between the quarks [11, 15]. It is consistent with a naive picture that the two constituent quarks are connected by a flux tube with a constant tension. At short distance, the potential is expected to be Coulomb-like; this portion of the potential, however, is not expected to play a significant role [16]. Also, there is an electromagnetic hyperfine splitting which violates the relation 6, but its effect is also much smaller than the hyperfine splitting due to strong interaction [17].

Apart from the theoretical importance, the above mass difference indicates that B^* cannot decay to $B\pi$. It has a practical implication that one cannot tag the sign of the bottom flavor by the decays such as $B^{*+} \rightarrow B^0\pi^+$ where the charge sign of the pion tells us if the neutral B meson is bottom or anti-bottom. Such flavor tagging would have made it easy to study the CP violating decay asymmetry in B^0 or $\bar{B}^0 \rightarrow \psi K_S, \pi^+\pi^-$

etc. particularly in hadron colliders. Now we have to hope that there may be a higher resonance that decays to $B\pi$ which is narrow and produced copiously [18].

3 Non-Suppressed Decays

3.1 *Effective Hamiltonian*

The interaction of interest for B meson decays comes from the charged current part of the Standard Model Lagrangian [19]:

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}(u, c, t)_L \gamma_\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^\mu. \quad (7)$$

where g is the weak coupling constant, the subscript L for the quark field indicates left-handed component (e.g. $u_L = \frac{1}{2}(1 - \gamma_5)u$ etc.), and the matrix V is the Cabbibo-Kobayashi-Masukawa (CKM) matrix:

$$V \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (8)$$

The experimental value of the CKM matrix V is well represented by [20, 21] (assuming unitarity of V)

$$V \sim \begin{pmatrix} 1 & \lambda & |V_{ub}|e^{i\alpha} \\ -\lambda & 1 & \lambda^2 \\ |V_{td}|e^{i\beta} & -\lambda^2 & 1 \end{pmatrix} \quad \text{where} \quad \begin{cases} \lambda \sim 0.22 \\ \alpha = \arg(V_{ub}) \\ \beta = \arg(V_{td}) \end{cases} \quad (9)$$

and the magnitude of V_{ub}, V_{td} is of order λ^3 . Taking the first and third columns, the unitarity condition

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0 \quad (10)$$

becomes a triangle as below (called the unitary triangle).

$$(11)$$

At energy scales well below the W mass, the propagation of W can be ‘integrated out’ and we obtain 4-fermion effective Hamiltonian [22] relevant to B decays given by

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} (C_1(\mu) \mathbf{O}_1 + C_2(\mu) \mathbf{O}_2) + \dots \quad (12)$$

$$\mathbf{O}_1 = (\bar{d}u)(\bar{c}b), \quad \mathbf{O}_2 = (\bar{c}u)(\bar{d}b) \quad (13)$$

where $G_F = g^2/(4\sqrt{2}M_W^2)$ is the Fermi coupling constant and the quark current $(\bar{q}'q)$ is a short hand for $\bar{q}'_\alpha \gamma_\mu (1 - \gamma_5) q_\alpha$ which is a color-singlet $V - A$ current (α : color index). Any combination of replacements $c \rightarrow u, u \rightarrow c$ and $d \rightarrow s$ can be made to obtain other possible interactions as long as the replacements are consistently made including the indexes of the CKM matrix elements. The terms shown in (12) are part of an expansion of the effective hamiltonian (the operator product expansion [23]). It has an advantage that the calculable short-distance effects are separated into the coefficients of the operators (Wilson coefficients) while the long distance effects such as the state of valence quarks in mesons are absorbed into matrix elements of the operators.

Without QCD correction, we only have the first operator \mathbf{O}_1 which is shown diagrammatically in Figure 3(a). With QCD correction, gluons flying between the quark lines can shuffle the color flows and generate an effective neutral current operator \mathbf{O}_2 shown in Figure 3(b). The Wilson coefficients $C_{1,2}$ can be calculated using the leading-logarithm approximation (LLA) [24]

$$C_1 = \frac{1}{2}(C_+ + C_-) \quad C_2 = \frac{1}{2}(C_+ - C_-) \quad (14)$$

with

$$C_\pm = \left[\frac{\alpha_S(\mu^2)}{\alpha_S(M_W^2)} \right]^{\frac{d_\pm}{2b}} \quad (15)$$

where $d_- = -2d_+ = 8$, and α_S is the running coupling constant of strong interaction given by

$$\alpha_S(\mu^2) = \frac{4\pi}{b \log(\mu^2/\Lambda_{\text{QCD}}^2)} \quad \text{with} \quad b = 11 - \frac{2}{3}n_f. \quad (16)$$

with n_f being the number of relevant flavors, and μ the typical mass scale of problems in question. Note that C_+ and C_- are related by $C_+^2 C_- = 1$. With $\mu = m_b = 5$ GeV, $n_f = 4$, and $\Lambda_{\text{QCD}} = 0.25$ GeV we have

$$C_1(m_b) = 1.11 \quad C_2(m_b) = -0.26. \quad (17)$$

The next-to-leading logarithm approximation (NLLA) has been computed [25]; the result does not differ drastically from the LLA result quoted above. For the transition $b \rightarrow cs\bar{c}$, however, the momentum transfer associated with the light quarks are much smaller than the bottom mass scale and as a result the corresponding coefficients could be significantly

Figure 3: Four fermion operators of the effective Hamiltonian responsible for B meson decays.

different from (17). In fact, in one estimation using heavy quark effective theory (HQET) [31], the coefficients are about 30% larger for C_1 and almost twice as large for C_2 [32]:

$$C_1 \sim 1.45 \quad C_2 \sim -0.45 \quad (\text{for } b \rightarrow cs\bar{c}) \quad (18)$$

There are also 4-fermion operators of the type shown in Figure 3(c) called Penguin operators [26]. The corresponding coefficients, however, are small and the Penguin operators are relevant only for highly suppressed decays such as $B \rightarrow K^*\gamma$ and $K\pi$, to which we will come back later.

3.2 *Two-body Decays and Factorization*

Compared to semileptonic decays, hadronic decays are harder to understand due to variety of short and long-distance strong interactions among the quarks involved. Two-body hadronic decays, however, are the simplest kind, and some framework of understanding - factorization - exists [27]. Also, it should be noted that two-body decays account for a substantial fraction of total hadronic decays of heavy mesons ($\sim 15\%$ for bottom mesons and $\sim 75\%$ for charm mesons when resonances are included [28]).

The idea of factorization for hadronic weak decay dates back at least to the early 60's when Schwinger showed that the $\Delta I = 3/2$ transition of $K \rightarrow \pi\pi$ can be estimated from the corresponding semileptonic rate [29]. The procedure, however, was not considered to be accurate; in fact, when Feynman reported calculations of $\Lambda \rightarrow p\pi$ and $K^+ \rightarrow \pi^+\pi^0$

Figure 4: Decay $\overline{B}^0 \rightarrow D^+\pi^-$ by the operator \mathbf{O}_1 (a) and \mathbf{O}_2 (b). The latter is suppressed by a factor ξ .

using the idea of factorization [30], he preceded the discussion by the following disclaimer: ‘You may not wish to consider this line of flimsy reasoning; we are becoming very uncertain about this matter, nevertheless I shall present it.’ There is, however, a good reason to believe that the factorization works well for certain B decays.

We take $\overline{B}^0 \rightarrow D^+\pi^-$ as an example. This can occur by the operator \mathbf{O}_1 as shown in Figure 4 (a), where it is assumed that the $B \rightarrow D$ transition is caused by the current operator $(\overline{c}b)$ and that π^- is created by the current operator $(\overline{d}u)$. Assuming that the $B \rightarrow D$ transition and the π^- creation are independent, the amplitude can be written as

$$\langle D^+\pi^- | (\overline{d}u)(\overline{c}b) | \overline{B}^0 \rangle = \langle \pi^- | (\overline{d}u) | 0 \rangle \langle D^+ | (\overline{c}b) | \overline{B}^0 \rangle \quad (19)$$

which constitutes the essence of the factorization assumption.

It is instructive to visualize the situation intuitively. A B meson may be viewed as an analog of a hydrogen atom where the heavy bottom quark is at the center surrounded by a cloud made of light quark and gluon [Figure 5(a)]. Upon the decay of the b quark, the b quark disappears and c , \overline{u} , and d quarks appear. The c quark will combine with the original cloud that was around the b quark to form a D meson, and the $\overline{u}d$ pair will eventually turn into a pion. Here one can cast doubts on the factorization assumption on two points:

1. When the $\overline{u}d$ pair passes through the cloud, it may strongly interact with the cloud, in which case the formation of the D meson and the creation of the pion cannot be

Figure 5: An intuitive picture of the decay $\overline{B}^0 \rightarrow D^+ \pi^-$. Before the decay (a), immediately after the b quark decay (b), and right after the formation of final state mesons (c).

independent.

2. After the D meson and the pion are formed, they may re-scatter through final-state interaction (FSI); e.g. $D^+ + \pi^- \rightarrow D^0 + \pi^0$ etc.

For each of the above, Bjorken has argued that it does not pose serious problem for the factorization assumption [33]. First, the invariant mass of the $\overline{u}d$ pair is of order pion mass; thus, they are highly collinear and close together. Since the total color of the pair is zero, they form a small color dipole and the cloud cannot see them from some distance away. The pair is thus expected to pass through the cloud without much interaction. Second, the formation time of the pion in its own rest frame is of order 0.3 fm/c which is the time for light to propagate from the center of the pion to the edge. Since the pion is highly energetic (~ 2.5 GeV), by the time it is formed the distance between the D meson and the pion is already several fermis; thus, they cannot interact through FSI. A similar argument of ‘color transparency’ was also used for production of ρ and ψ in high energy scatterings [34].

This line of argument has been put forward by Dugan and Grinstein in the framework of QCD and the heavy quark effective theory, and it has been shown that factorization holds in the limit of $M_{B,D} \rightarrow \infty$ while M_B/M_D is kept constant [35]. For decays which involve two charmed mesons such as $\overline{B}^0 \rightarrow D_S^- D^+$, the two mesons in the final state are

partially overlapped at the formation time, and thus the factorization may not work well for these decays. Factorization is known to hold also for the large N_C limit where N_C is the number of colors [36]. Even though the correction to the limit is of order $1/3$ which is quite large, the applicability of the $1/N_C$ argument is not restricted to the large velocity limit [37], and thus complementary to the ‘color transparency’ argument.

The decay $\overline{B}^0 \rightarrow D^+ \pi^-$ can also proceed by the operator \mathbf{O}_2 as shown in Figure 4(b). In this case, naively only the color singlet component of the \overline{u} and d legs is expected to contribute. Applying Fierz transformations to color indexes as well as to gamma matrices [38], \mathbf{O}_2 can be written as

$$\mathbf{O}_2 = \frac{1}{3}\mathbf{O}_1 + \frac{1}{2}(\overline{d}\lambda^i u)(\overline{c}\lambda_i b) \quad (20)$$

where the second term is a color singlet operator formed by two color-octet currents with λ^i being the $SU(3)$ Gell-Mann matrices. Thus, \mathbf{O}_2 contains \mathbf{O}_1 within itself, and consequently \mathbf{O}_1 and \mathbf{O}_2 are *not* orthogonal [37]. The overall coefficient of \mathbf{O}_1 is then $C_1 + C_2/3$. For the decay $\overline{B}^0 \rightarrow D^0 \pi^0$, the relevant operator is \mathbf{O}_2 . There, the role of \mathbf{O}_1 and \mathbf{O}_2 are inverted with the overall coefficient of \mathbf{O}_2 being $C_2 + C_1/3$. In fact, we can write (12) in two ways

$$\begin{aligned} C_1\mathbf{O}_1 + C_2\mathbf{O}_2 &= (C_1 + \frac{C_2}{3})\mathbf{O}_1 + \frac{1}{2}(\overline{d}\lambda^i u)(\overline{c}\lambda_i b) \\ &= (C_2 + \frac{C_1}{3})\mathbf{O}_2 + \frac{1}{2}(\overline{d}\lambda^i b)(\overline{c}\lambda_i u) \end{aligned} \quad (21)$$

Assuming factorization, the effective Hamiltonian may then be written in terms of ‘factorized hadron operators’ [39] as

$$\mathcal{H}_{\text{had}} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} [a_1 (\overline{d}u)_{\text{had}} (\overline{c}b)_{\text{had}} + a_2 (\overline{d}b)_{\text{had}} (\overline{c}u)_{\text{had}}] \quad (22)$$

where the above arguments suggest

$$\begin{aligned} a_1 &= C_1 + \xi C_2 \\ a_2 &= C_2 + \xi C_1 \end{aligned} \quad \text{with} \quad \xi = \frac{1}{3}, \quad (23)$$

where the effect of \mathbf{O}_2 to the first term and that of \mathbf{O}_1 to the second term is parametrized by ξ (sometimes called ‘color suppression factor’). The contribution of the octet current term in (20), however, may have a significant effect; in fact, an estimation using QCD sum rule indicates that its contribution may in effect lead to $\xi \sim 0$ [40]. Also, an analysis of charm decays suggests ξ near zero [28]. It has thus been suggested that a_1, a_2 be taken as free parameters [28].

Given the factorized Hamiltonian (22), one can then write down the amplitude for a decay. For example, if X^- is a meson made of valence quarks d and \overline{u} ,

$$\text{Amp}(\overline{B}^0 \rightarrow D^+ X^-) = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} a_1 \langle X^- | (\overline{u}d)_{\text{had}}^\mu | 0 \rangle \langle D^+ | (\overline{c}b)_{\text{had}\mu} | \overline{B}^0 \rangle \quad (24)$$

where we have from Lorentz invariance

$$\langle X^- | (\bar{u}d)_{\text{had}}^\mu | 0 \rangle = -if_X q^\mu \quad (\text{for } X: \text{pseudo scalar}) \quad (25)$$

$$\langle X^- | (\bar{u}d)_{\text{had}}^\mu | 0 \rangle = f_X m_X \epsilon^\mu \quad (\text{for } X: \text{vector or axial vector}) \quad (26)$$

with f_X being a parameter of energy dimension (called the decay constant). The current matrix element is the same as that appears in the corresponding semileptonic decay [41] evaluated at $q^2 = m_\pi^2$:

$$\langle D^+ | (\bar{c}b)_{\text{had}\mu} | \bar{B}^0 \rangle = \left(P_B + P_D - \frac{m_B^2 - m_D^2}{q^2} q \right)_\mu F_1(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0(q^2) \quad (27)$$

where F_0 and F_1 are longitudinal and transverse form factors respectively [one can easily verify that the coefficient of F_1 satisfies $(\dots)_\mu q^\mu = 0$]. For the case of pion emission, the transverse component exactly vanishes (by definition) and we have

$$\text{Amp}(\bar{B}^0 \rightarrow D^+ \pi^-) = -i \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} a_1 f_\pi (m_B^2 - m_D^2) F_0(m_\pi^2). \quad (28)$$

The form factors $F_{0,1}$ may be either obtained from semileptonic decays or calculated by models such as the relativistic harmonic oscillator model together with the pole dominance [41]. They are relatively slowly varying functions of order 1. In addition, the heavy quark effective theory allows us to relate all form factors for transitions between heavy mesons to a universal form factor [42]. Similar procedures are applied to other decay modes.

In general, we may distinguish three classes of decays when we consider two-body decays of heavy mesons mediated by operators of the types $\mathbf{O}_{1,2}$ in spectator mode (i.e. the light quark in the parent meson does not participate in the weak decay) [43]:

Class 1 Only the first term in (22) contributes and the amplitude is proportional to a_1 ; e.g. $\bar{B}^0 \rightarrow D^+ \pi^-$.

Class 2 Only the second term in (22) contributes and the amplitude is proportional to a_2 ; e.g. $\bar{B}^0 \rightarrow D^0 \pi^0$. Sometimes called ‘color-suppressed’ decays.

Class 3 Both terms in (22) contribute and the amplitude contains both a_1 and a_2 ; e.g. $B^- \rightarrow D^0 \pi^-$.

Some comments are in order. If both final-state particles are charged, then it is Class 1, if both are neutral, then it is Class 2, if one is neutral and the other is charged, then it depends. In $\bar{B}^0 \rightarrow D^+ \pi^-$, the current $B \rightarrow D$ emits a π and thus the pion decay constant f_π is involved. In $\bar{B}^0 \rightarrow D^0 \pi^0$, the current $B \rightarrow \pi$ emits a D meson and thus the D meson decay constant f_D is involved. In $B^- \rightarrow D^0 \pi^-$, a class 1 amplitude and a class 2 amplitude interfere and thus both f_π and f_D are involved. Also, note that in $\bar{B}^0 \rightarrow D^0 \pi^0$, the ‘color transparency’ argument does not apply since the color-singlet pair

Figure 6: The annihilation and W -exchange processes.

passing through the cloud is now $c\bar{u}$ pair which are moving quite slowly, and it may form a D meson before leaving the cloud. Thus, factorization may not be a good assumption in this case.

Heavy mesons may also decay through valence quark annihilation or W -exchange processes [44] as shown in Figure 6 which are also mediated by interactions of types $\mathbf{O}_{1,2}$. Such processes have been discussed in the context of the lifetime difference between D^+ and D^0 , but thought to be helicity-suppressed [45], and also suppressed by form factor effect when two-body decays are considered [46]. It was suggested, however, that the helicity suppression may be lifted when soft gluon effects are taken into account [47]. Even though annihilation/exchange processes are usually ignored in B decays, it has not been proven that they do not significantly contribute in all types of decays.

3.3 *Experimental Test of Factorization*

The decays $B \rightarrow PP, PV$ have definite final spin state, where P is a pseudo scalar meson and V a vector meson, thus the decay rate is the only dynamical parameter that can be tested. On the other hand, the decays $B \rightarrow VV$ has three possible helicity amplitudes which can also be compared against prediction of factorization.

For the test of decay rates, we take $\bar{B}^0 \rightarrow D^{*+}X^-$ with X^- being π^- , ρ^- , or a_1^- . As described above, factorization allows us to estimate the decay rates of these modes from the q^2 -dependent form factors of the corresponding semileptonic decay $\bar{B}^0 \rightarrow D^{*+}l^-\nu$. In other words, there is a simple relation between the differential decay rate of

the semileptonic mode at $q^2 = m_X^2$ and the corresponding non-leptonic decay rate, which can be conveniently written as [33]

$$R \stackrel{\text{def}}{=} \frac{Br(\bar{B}^0 \rightarrow D^{*+} X^-)}{\left. \frac{dBr}{dq^2}(\bar{B}^0 \rightarrow D^{*+} l^- \nu) \right|_{q^2=m_X^2}} = 6\pi^2 f_X^2 |V_{ud}|^2 \quad (29)$$

where f_X is the decay constant of the meson X . No QCD correction is included in the expression on the right hand side [49]. If QCD correction is to be included, a reasonable choice would be to add $(C_1 + C_2/3)^2$ to the right hand side of (29). This is because, in (21), the contribution from the octet current has been shown to be suppressed in the decays in question as shown by Dugan and Grinstein [35]. However, $C_1 + C_2/3$ is unity to the first order due to the relation $C_+^2 C_1 = 1$; thus, we will proceed without QCD correction. The above formula is applicable for X being any spin-1 particle or any light spin-0 particle (assuming factorization, of course) [48]. When the particle X is spin-0, it cannot replace all the helicity degrees of freedom of the D^* appearing in the semileptonic decay, and the formula is correct only in the limit of $m_X \ll m_B$. The correction for pion, however, is negligible ($\sim 0.5\%$). If X is spin-1, then no such restriction applies. If D^* is replaced by D , then a similar helicity projection factor should be included.

The procedure of the test is to measure the decay rate $B \rightarrow D^* X$ and the differential semileptonic rate $d\Gamma/dq^2$ to obtain the ratio R , and then compare it to the value expected from factorization: $6\pi^2 f_X^2 |V_{ub}|^2$. The q^2 distribution of the semileptonic decay $\bar{B} \rightarrow D^{*+} l^- \nu$ is shown in Figure 7, which is a combination of ARGUS [50] and CLEO [51] data. The shape is fit to three different models [41, 52, 53] to obtain the value at given q^2 .

The decay constants can be obtained by the leptonic decay rate [54]

$$\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu) = \frac{G_F^2 f_\pi^2}{8\pi} m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2, \quad (30)$$

for pion which gives $f_\pi = 132$ MeV. From the tau decay rates

$$\Gamma(\tau^- \rightarrow V^- \bar{\nu}_\tau) = \frac{m_\tau^3}{16\pi} G_F^2 |V_{ud}|^2 f_V^2 \left(1 - \frac{m_V^2}{m_\tau^2}\right)^2 \left(1 + 2\frac{m_V^2}{m_\tau^2}\right) \quad (31)$$

where V is a vector or axial vector, we get $f_\rho = 197 \pm 3$ MeV and $f_{a_1} = 178 \pm 28$ MeV. Including the effect of decay width of meson [56], these go up to $f_\rho = 210 \pm 3$ MeV and $f_{a_1} = 201 \pm 32$ MeV. Using the isospin symmetry relation $f_{\rho^-} = f_{\rho^0}$, the decay constant of ρ can also be obtained from $\Gamma(\rho^0 \rightarrow e^+ e^-)$ measured in $e^+ e^- \rightarrow \rho^0$ by

$$\Gamma(V^0 \rightarrow e^+ e^-) = \frac{4\pi\alpha^2}{3m_V} c_V f_V^2 \quad (32)$$

where $c_V = 1/2$ for ρ [55]; this gives $f_\rho = 216 \pm 5$ MeV which we will use.

Table 3 summarizes the result of the comparison. Note that in taking the ratio (29), uncertainty in D^* detection efficiency is canceled. This of course assumes that same D^*

Figure 7: The distribution of the lepton-neutrino invariant mass (q^2) for the process $\overline{B} \rightarrow D^{*+}l^-\nu$ as measured by CLEO and ARGUS.

and D branching ratios are used for the measurements of $D^*l\nu$ mode and D^*X mode; a correction has been made to the values of Figure 7 using the new measurements from CLEO [57, 58]. The agreement is quite good for π and ρ . For a_1 , the measured R is about a factor of two larger than the expected value, but statistically it is only 1.5 sigma's. This could well be due to breakdown of factorization at a_1 mass of 1.26 GeV. The branching ratio of D^*a_1 is determined assuming that the $D^{*+}\pi^+\pi^-\pi^-$ final state with $1.0 < M_{3\pi} < 1.6$ GeV is dominated by a_1 . Figure 8 shows the 3π invariant mass distribution for the decay mode $\overline{B}^0 \rightarrow D^{*+}\pi^+\pi^-\pi^-$. The a_1 peak is clearly seen, and amount of non a_1 contribution is quite small.

As stated earlier, for $B \rightarrow VV$ decays there are helicity degrees of freedom which cannot be uniquely determined by kinematics. The factorization assumption leads to specific prediction for helicity amplitudes which can then be tested experimentally. For example, once the matrix element is factorized as

$$Amp(B \rightarrow D^*\rho) \propto \langle \rho | (\overline{u}d)_{\text{had}}^\mu | 0 \rangle \langle D^* | (\overline{c}b)_{\text{had}_\mu} | \overline{B} \rangle, \quad (33)$$

then by Lorentz invariance the rho production part $\langle \rho | (\overline{u}d)_{\text{had}}^\mu | 0 \rangle$ is proportional to the ρ polarization vector ϵ^μ [see (26)]. It then acts the same way as the polarization vector of W in semileptonic decay resulting in the same ρ polarization as that of the W in semileptonic decay at $q^2 = m_\rho^2$. If factorization is not valid, this argument cannot hold, and thus it serves as a good check of factorization.

The polarization of ρ can be measured by the distribution of $\rho^- \rightarrow \pi^0 \pi^-$ polar decay angle θ_ρ in the ρ rest frame with respect to the D^* direction in the same frame. Longitudinal polarization (helicity=0) would have $\cos^2 \theta_\rho$ distribution while transverse polarization (helicity= ± 1) would have $\sin^2 \theta_\rho$ distribution. Or equivalently, one can measure the decay angle of D^* (θ_D) in the same way since the helicity of D^* is the same as that of ρ . In fact, the angular distribution can be written as

$$\frac{d\Gamma}{d\cos\theta_\rho d\cos\theta_D} \propto \frac{1-a_L}{4} \sin^2 \theta_\rho \sin^2 \theta_D + a_L \cos^2 \theta_\rho \cos^2 \theta_D \quad (34)$$

where a_L is the fraction of longitudinal polarization

$$a_L = \frac{|H_0|^2}{|H_+|^2 + |H_0|^2 + |H_-|^2} \quad (35)$$

with $H_{+,0,-}$ being the three helicity amplitudes. Figure 9 shows the distributions of θ_ρ and θ_D for data. A simultaneous fit to the two angles gives

$$a_L = 0.93 \pm 0.05 \pm 0.05 \quad (\text{CLEO}) \quad (36)$$

where the first error is statistical and the second systematic which includes uncertainty in background subtraction and detection efficiencies. The experimental measurement of the polarization in the semileptonic decay is unfortunately not available at this point, and we have to compare the above measurement to absolute theoretical prediction which requires some assumption on form factors. One estimate using HQET [48] gives

$$a_L = 0.88 \quad (\text{factorization} + \text{HQET}) \quad (37)$$

which is in agreement with the data.

X	$Br(D^{*+}X^-)$ (%) ^(a)	dBr/dq^2 (%/GeV ²)	$R(\text{measured})$ (GeV ²)	f_X (GeV)	$R = 6\pi^2 V_{ud} ^2 f_X^2$ (GeV ²)
π	0.265 ± 0.036	0.23 ± 0.05	1.15 ± 0.30	0.132 ± 0.0005	0.98 ± 0.01
ρ	0.735 ± 0.106	0.25 ± 0.04	2.94 ± 0.63	0.216 ± 0.005	2.63 ± 0.12
a_1	1.32 ± 0.30 ^(b)	0.32 ± 0.04	4.13 ± 1.07	0.201 ± 0.032	2.27 ± 0.72

(a) The errors are statistical only.

(b) It is assumed that $D^{*+}a_1^-$ dominates $D^{*+}\pi^+\pi^-\pi^-$ mode where the 3π mass is between 1.0 and 1.6 GeV.

Table 3: Test of factorization. Branching fraction of $B \rightarrow D^*X$ is compared to the corresponding semileptonic decay evaluated at $q^2 = m_X^2$.

Figure 8: The 3π mass distribution in the decay $\overline{B}^0 \rightarrow D^{*+}\pi^+\pi^-\pi^-$. (a) Monte Carlo simulation for $\overline{B}^0 \rightarrow D^{*+}a_1^-, a_1^- \rightarrow \rho^0\pi^-$. (a) Monte Carlo simulation where $\rho^0\pi^-$ is uniform in phase space. (c) Data with B -mass side bands subtracted.

Figure 9: The angular distributions for D^* decay angle (a) and the ρ decay angle (b) in $\overline{B}^0 \rightarrow D^{*+}\rho^-$.

This helicity=0 dominance can be intuitively understood as follows (see the figure above). When the $\bar{u}d$ pair is emitted, they are nearly collinear, and the helicities are left-handed for the d quark and right-handed for the \bar{u} quark. Therefore the total helicity is zero which is transferred to the final ρ meson assuming that there is no final state interaction that changes the spin state. This feature is independent of specific choice of form factors, while it does assume factorization.

3.4 Extraction of a_1 and a_2

In this section, we will take the coefficients a_1 and a_2 as free parameters in the factorized effective Hamiltonian (22), and try to find their values by fitting to measured branching ratios. First, we will use $B \rightarrow D\pi$ and ψK decays to demonstrate the procedure, then a global fit to clean modes will be performed.

In order to extract a_1 and a_2 , we need the form factors of $B \rightarrow D$ transition. This is quite well known; we will use the result of the fit to the universal form factor under the framework of HQET. The relevant value here is $F_0(q^2 = m_\pi^2) = 0.58$. For the $B \rightarrow \pi$ or K transition, there is no experimental data, and a model calculation is used where the overlap of B and the light meson wave functions is obtained by relativistic harmonic oscillator model and the q^2 dependence is given by pole dominance. The coefficients of $a_{1,2}$ below are taken from Reference [27].

Class-I (determination of a_1): The decay amplitude of $\bar{B}^0 \rightarrow D^+\pi^-$ (or for any two-body decay $B \rightarrow PP$) is given by

$$\Gamma = \frac{p}{8\pi M_B^2} |Amp|^2 \quad (38)$$

where p is the momentum in the B rest frame. Using the factorized amplitude (28) together with $V_{cb} = 0.045$, $V_{ud} = 0.975$, $G_F = 1.166 \times 10^{-5}$ (GeV $^{-2}$), $F_0(m_\pi^2) = 0.58$, and $\tau_B = 1.18$ ps, we get

$$Br(D^+\pi^-) = 0.264a_1^2 \quad (\%). \quad (39)$$

The measured branching ratio is $Br(D^+\pi^-) = 0.29 \pm 0.04\%$ from CLEO, where the error is statistical only. It then gives $a_1 = 1.1$.

Class-II (determination of a_2): In $\bar{B}^0 \rightarrow D^0\pi^0$, D meson is emitted and the transition is from B to π . Proceeding the same way as before, we get

$$Br(D^0\pi^0) = 0.201 \left(\frac{f_D(\text{GeV})}{0.22} \right)^2 a_2^2 \quad (\%). \quad (40)$$

where the isospin factor 1/2 is included (π^0 is half $\bar{u}u$ and half $\bar{d}d$). Experimentally, only upper limit exists for this mode: a recent number from CLEO is $Br(D^0\pi^0) < 0.035\%(90\%C.L.)$, which corresponds to $|a_2| < 0.4$.

The decay $B^- \rightarrow \psi K^-$ is also a Cabbibo-favored Class-II decay. The M_B distribution by CLEO is shown in Figure 10(a) together with other related modes that are used

Figure 10: The beam-constrained masses for $B^- \rightarrow \psi K^-$ (a), $B^0 \rightarrow \psi K^0$ (b), $B^- \rightarrow \psi K^{*-}$ (c), and $B^0 \rightarrow \psi K^{*0}$ (d) (CLEO collaboration)

in the combined fit later; there are about 60 signal events with little background. Mass peaks for $B^- \rightarrow \psi K^-$ and ψK^{*0} by CDF are shown in Figure 11 [59]. The transition is $B \rightarrow K$ and ψ is emitted. The decay constant of ψ can be obtained from its e^+e^- width: $f_\psi = 384 \pm 14 \text{ MeV}$. The expected branching ratio is

$$Br(\psi K^-) = 1.819 a_2^2 \quad (\%) \quad (41)$$

where the large coefficient is primarily due to the large decay constant of ψ . The measurement $Br(\psi K^-) = 0.110 \pm 0.015$ (CLEO) gives $|a_2| = 0.26$. One point of caution is that a_2 in $b \rightarrow c\bar{c}s$ transition is likely to be different from a_2 in $b \rightarrow c\bar{u}d$ transition. In fact, the values of $C_{1,2}$ themselves are expected to be different as seen in (18). Nonetheless, they are often assumed to be the same and we will proceed with this assumption for now.

Class-III (determination of a_2/a_1): As stated earlier, for Class-II and Class-III decays, the factorization assumption is not well founded. However, if we assume the factorized Hamiltonian (22), we can obtain the sign as well as the absolute value of a_2/a_1 through the interference of the two types of diagrams shown in Figure 4. For example, the branching fraction of $B^- \rightarrow D^0\pi^-$ (normalized to $\bar{B}^0 \rightarrow D^+\pi^-$) is given by

$$\frac{Br(D^0\pi^-)}{Br(D^+\pi^-)} = \left[1 + 1.230 \frac{a_2}{a_1} \left(\frac{f_D(\text{MeV})}{220} \right) \right]^2. \quad (42)$$

Figure 11: Invariant mass peaks for $B^- \rightarrow \psi K^-$ (a) and $B^0 \rightarrow \psi K^{*0}$ (b) by the CDF collaboration.

The ratio measured by CLEO is $1.84 \pm 0.24 \pm 0.29$, and this leads to $a_2/a_1 = 0.29 \pm 0.11$. The positive sign is a direct consequence of $Br(D^0\pi^-) > Br(D^+\pi^-)$.

Tables 4-6 summarize measurements and expected branching ratios from the factorization model as calculated in Reference [27]. The agreements are excellent in all cases.

In order to obtain more accurate value for a_1 we fit four Class-I modes, $\bar{B}^0 \rightarrow D^+\pi^-, D^+\rho^-, D^{*+}\pi^-,$ and $D^{*+}\rho^-$. For a_2 , we use the Class-II modes $\bar{B}^0 \rightarrow \psi K^0, \psi K^{*0}$ and $B^- \rightarrow \psi K^-, \psi K^{*-}$. The result is

$$|a_1| = 1.15 \pm 0.04 \pm 0.04 \pm 0.09, \quad |a_2| = 0.26 \pm 0.01 \pm 0.01 \pm 0.02 \quad (43)$$

where the first error is statistical, the second and the third are systematic. The third error is due to the uncertainty in the ratio of production and that of lifetimes of charged vs neutral B mesons. The relevant quantity is $(f_+\tau_+)/ (f_0\tau_0)$ where f_+, f_0 are the production fractions and τ_+, τ_0 are the lifetimes. This value is sometimes assumed to be unity. A measurement from $Br(B^- \rightarrow D^{*0}l\nu)/Br(\bar{B}^0 \rightarrow D^{*+}l\nu)$ [62] is

$$\frac{f_+\tau_+}{f_0\tau_0} = 1.2 \pm 0.20 \pm 0.10 \pm 0.16. \quad (\text{CLEO}). \quad (44)$$

For determination of a_2/a_1 , we use the following four ratios of branching fractions: $B(D^0\pi^-)/B(D^+\pi^-)$, $B(D^0\rho^-)/B(D^+\rho^-)$, $B(D^{*0}\pi^-)/B(D^{*+}\pi^-)$, and $B(D^{*0}\rho^-)/B(D^{*+}\rho^-)$ to obtain

$$\frac{a_2}{a_1} = 0.23 \pm 0.04 \pm 0.03 \pm 0.10 \quad (45)$$

\bar{B}^0	CLEO (%)	ARGUS (%)	Model (%) [27]	$a_1 = 1.15$
$D^+\pi^-$	$0.29 \pm 0.04 \pm 0.03 \pm 0.05^a$	$0.48 \pm 0.11 \pm 0.11^d$	$0.264a_1^2$	0.35
$D^+\rho^-$	$0.81 \pm 0.11 \pm 0.12 \pm 0.13^a$	$0.9 \pm 0.5 \pm 0.3^d$	$0.621a_1^2$	0.82
$D^{*+}\pi^-$	$0.26 \pm 0.03 \pm 0.03 \pm 0.01^a$	$0.28 \pm 0.09 \pm 0.06^d$	$0.254a_1^2$	0.34
$D^{*+}\rho^-$	$0.74 \pm 0.10 \pm 0.13 \pm 0.03^a$	$0.7 \pm 0.3 \pm 0.3^d$	$0.702a_1^2$	0.93
$D^{*+}a_1^{-e}$	$1.26 \pm 0.20 \pm 0.14 \pm 0.04^a$		$0.97a_1^2(f_{a_1}/0.22)^2$	1.28
$D_{(2460)}^{*+}\pi^-$	$< 0.18^a$			
$D^+D_s^-$	1.2 ± 0.7^b	$1.7 \pm 1.3 \pm 0.6^c$	$1.213a_1^2(f_{D_s}/0.28)^2$	1.60
$D^+D_s^{*-}$		$2.7 \pm 1.7 \pm 0.9^c$	$0.859a_1^2(f_{D_{s^*}}/0.28)^2$	1.14
$D^{*+}D_s^-$	2.4 ± 1.4^b	$1.4 \pm 1.0 \pm 0.3^c$	$0.824a_1^2(f_{D_s}/0.28)^2$	1.09
$D^{*+}D_s^{*-}$		$2.6 \pm 1.4 \pm 0.6^c$	$2.203a_1^2(f_{D_{s^*}}/0.28)^2$	2.91
B^-				
$D^0D_s^-$	2.9 ± 1.3^b	$2.4 \pm 1.2 \pm 0.4^c$	$1.215a_1^2(f_{D_s}/0.28)^2$	1.61
$D^0D_s^{*-}$		$1.6 \pm 1.2 \pm 0.3^c$	$0.862a_1^2(f_{D_{s^*}}/0.28)^2$	1.14
$D^{*0}D_s^-$		$1.3 \pm 0.9 \pm 0.2^c$	$0.828a_1^2(f_{D_s}/0.28)^2$	1.10
$D^{*0}D_s^{*-}$		$3.1 \pm 1.6 \pm 0.5^c$	$2.206a_1^2(f_{D_{s^*}}/0.28)^2$	2.92

a. Preliminary result to be submitted to Phys. Rev. D. The first error is statistical, the second systematic, and the third error is due to uncertainties of D branching ratios.

b. Reference [60], $Br(D_S^+ \rightarrow \phi\pi^+) = 2\%$ is used.

c. Reference [61], $Br(D_S^+ \rightarrow \phi\pi^+) = 2.7\%$ is used.

d. Reference [3].

e. All events with 3π mass between 1.0 and 1.6 GeV (after background subtraction) are assumed to be a_1 .

Table 4: Class-I Branching ratios

\overline{B}^0	CLEO (%)	ARGUS (%)	Model (%) [27]	$a_1 = 0.26$
$D^0\pi^0$	$< 0.035^a$		$0.201a_2^2(f_D/0.22)^2$	0.014
$D^0\rho^0$	$< 0.042^a$		$0.136a_2^2(f_D/0.22)^2$	0.009
$D^{*0}\pi^0$	$< 0.072^a$		$0.213a_2^2(f_{D^*}/0.22)^2$	0.014
$D^{*0}\rho^0$	$< 0.092^a$		$0.223a_2^2(f_{D^*}/0.22)^2$	0.015
$D^0\eta$	$< 0.075^a$			
$D^0\eta'$	$< 0.074^a$			
$D^0\omega$	$< 0.048^a$			
$D^{*0}\eta$	$< 0.086^a$			
$D^{*0}\eta'$	$< 0.36^a$			
$D^{*0}\omega$	$< 0.13^a$			
$\psi\overline{K}^0$	$0.075 \pm 0.024 \pm 0.008^a$	$0.08 \pm 0.06 \pm 0.02^b$	$1.817a_2^2$	0.123
$\psi\overline{K}^{*0}$	$0.169 \pm 0.031 \pm 0.018^a$	$0.11 \pm 0.05 \pm 0.02^b$	$2.927a_2^2$	0.198
$\psi'\overline{K}^0$	$< 0.08^a$	$< 0.28^b$	$1.065a_2^2$	0.072
$\psi'\overline{K}^{*0}$	$< 0.19^a$	$< 0.23^b$	$1.965a_2^2$	0.133
$\chi_{c1}\overline{K}^0$	$< 0.27^a$			
$\chi_{c1}\overline{K}^{*0}$	$< 0.21^a$			
B^-				
ψK^-	$0.110 \pm 0.015 \pm 0.009^a$	$0.07 \pm 0.03 \pm 0.01^b$	$1.819a_2^2$	0.123
ψK^{*-}	$0.178 \pm 0.051 \pm 0.023^a$	$0.16 \pm 0.11 \pm 0.03^b$	$2.932a_2^2$	0.198
$\psi' K^-$	$0.061 \pm 0.023 \pm 0.015^a$	$0.18 \pm 0.08 \pm 0.04^b$	$1.068a_2^2$	0.072
$\psi' K^{*-}$	$< 0.30^a$	$< 0.49^b$	$1.971a_2^2$	0.133
$\chi_{c1}K^-$	$0.097 \pm 0.040 \pm 0.009^a$			
$\chi_{c1}K^{*-}$	$< 0.21^a$			

a. Preliminary result to be submitted to Phys. Rev. D.

b. Reference [3]. Modes involving a K_s are multiplied by two to obtain the branching ratios for \overline{K}^0 .

Table 5: Class-II Branching ratios

B^-	CLEO (%)	ARGUS (%)	Model (%) [27]	$a_1 = 1.15$ $a_2 = 0.26$
$D^0\pi^-$	0.55 ± 0.04 $\pm 0.03 \pm 0.02^a$	$0.20 \pm 0.08 \pm 0.06^b$	$0.265(a_1 + 1.230a_2(f_D/0.22))^2$	0.57
$D^0\rho^-$	1.35 ± 0.12 $\pm 0.12 \pm 0.04^a$	$1.3 \pm 0.4 \pm 0.4^b$	$0.622(a_1 + 0.662a_2(f_D/0.22))^2$	1.09
$D^{*0}\pi^-$	0.49 ± 0.07 $\pm 0.06 \pm 0.03^a$	$0.40 \pm 0.14 \pm 0.12^b$	$0.255(a_1 + 1.292a_2(f_{D^*}/0.22))^2$	0.56
$D^{*0}\rho^-$	1.68 ± 0.21 $\pm 0.22 \pm 0.08^a$	$1.0 \pm 0.6 \pm 0.4^b$	$0.703[a_1^2 + 0.635a_2^2(f_{D^*}/0.22)^2$ $+ 1.487a_1a_2(f_{D^*}/0.22)]$	1.27
$D^{*0}a_1^-^c$	1.88 ± 0.40 $\pm 0.30 \pm 0.10^a$			
$D_{(2420)}^{*0}\pi^-$	0.11 ± 0.05 $\pm 0.04 \pm 0.03^a$			
$D_{(2460)}^{*0}\pi^-$	$< 0.15^a$			
$D_{(2420)}^{*0}\rho^-$	$< 0.14^a$			
$D_{(2460)}^{*0}\rho^-$	$< 0.5^a$			

a. Preliminary result to be submitted to Phys. Rev. D. The first error is statistical, the second systematic, and the third error is due to uncertainties of D branching ratios.

b. Reference [3].

c. All events with 3π mass between 1.0 and 1.6 GeV (after background subtraction) are assumed to be a_1 .

Table 6: Class-III Branching ratios

where $(f_+\tau_+)/ (f_0\tau_0) = 1.2$ is used and the last error is due to the uncertainty in this quantity. The absolute value of a_2/a_1 is consistent with the value obtained above which is $0.26/1.15 = 0.23$, and the negative sign seems to be excluded. From (23), we have

$$\frac{a_2}{a_1} = \frac{C_1 + \xi C_2}{C_2 + \xi C_1} \quad \rightarrow \quad \xi = \frac{a_2/a_1 - C_2/C_1}{1 - (C_2/C_1)(a_2/a_1)}. \quad (46)$$

Using $C_1 = 1.11$, $C_2 = -0.26$, the negative value $a_2/a_1 = -0.23$ corresponds to $\xi = 0.01$ and the positive value $a_2/a_1 = 0.23$ corresponds to $\xi = 0.44$. Thus, $\xi = 0$ as suggested by an analysis of charm decays [28] seems to be excluded in the B decays. However, one has to keep in mind that in the analysis above, the factorization was applied to questionable cases where emitted meson is heavy. Also, the factorization is not expected to hold well for charm decays, so the formulation using $a_{1,2}$ itself is in question in charm decays.

So far only Class-II modes observed are for $b \rightarrow c\bar{c}s$ only. As one can see from the table, however, the present sensitivity is close to the expected values for the $D^0\pi^0$ and related modes. It is likely that these modes will be observed soon.

3.5 Final State Interaction

The factorization assumes that effect of final state interaction is negligible. Therefore any test that is sensitive to final state interaction is also a test of factorization.

One way is to perform an isospin analysis on a set of isospin-related modes. For example, the relevant Hamiltonian for $B \rightarrow D\pi$ decays has isospin structure $|I, I_z\rangle = |1, -1\rangle$ (i.e. $b \rightarrow c\bar{u}d$ - simply a creation of $\bar{u}d$ pair as long as isospin is concerned). Separating the Hamiltonian to an isospin violating part S and an isospin conserving part h , and treating S as if it is a particle (spurion), we can write, for example

$$\begin{aligned} S\bar{B}^0 &= \sqrt{\frac{1}{3}}|3/2, -1/2\rangle - \sqrt{\frac{2}{3}}|1/2, -1/2\rangle \\ D^+\pi^- &= \sqrt{\frac{1}{3}}|3/2, -1/2\rangle - \sqrt{\frac{2}{3}}|1/2, -1/2\rangle. \end{aligned} \quad (47)$$

Applying similar isospin decomposition to SB^- , $\bar{B}^0 \rightarrow D^0\pi^0$, and $B^- \rightarrow D^0\pi^-$, we get

$$\begin{aligned} \text{Amp}(S\bar{B}^0 \rightarrow D^+\pi^-) &= \frac{1}{3}\langle 3/2, -1/2|h|3/2, -1/2\rangle + \frac{2}{3}\langle 1/2, -1/2|h|1/2, -1/2\rangle \\ \text{Amp}(S\bar{B}^0 \rightarrow D^0\pi^0) &= \frac{\sqrt{2}}{3}\langle 3/2, -1/2|h|3/2, -1/2\rangle - \frac{\sqrt{2}}{3}\langle 1/2, -1/2|h|1/2, -1/2\rangle \\ \text{Amp}(SB^- \rightarrow D^0\pi^-) &= \langle 3/2, -3/2|h|3/2, -3/2\rangle \end{aligned} \quad (48)$$

Because of isospin invariance of h , the matrix elements depend only on the magnitude of the isospin: $\langle 3/2, -3/2|h|3/2, -3/2\rangle = \langle 3/2, -1/2|h|3/2, -1/2\rangle$ which we define to be

$\sqrt{3}A_{\frac{3}{2}}$. Together with a definition $\langle 1/2, -1/2 | h | 1/2, -1/2 \rangle = \sqrt{\frac{3}{2}}A_{\frac{1}{2}}$, we then obtain

$$\begin{aligned} \text{Amp}(D^+\pi^-) &= \sqrt{\frac{1}{3}}A_{\frac{3}{2}} + \sqrt{\frac{2}{3}}A_{\frac{1}{2}} \\ \text{Amp}(D^0\pi^0) &= \sqrt{\frac{2}{3}}A_{\frac{3}{2}} - \sqrt{\frac{1}{3}}A_{\frac{1}{2}} \\ \text{Amp}(D^0\pi^-) &= \sqrt{3}A_{\frac{3}{2}} \end{aligned} \tag{49}$$

where $A_{\frac{3}{2}}$ and $A_{\frac{1}{2}}$ are the isospin 3/2 and 1/2 amplitudes respectively. There are three unknown parameters: $|A_{\frac{3}{2}}|$, $|A_{\frac{1}{2}}|$, and $\delta = \arg(A_{\frac{3}{2}}/A_{\frac{1}{2}})$. Since there are three measurements of decay rates, one can solve for the three unknowns. Then the non-zero phase δ signifies the existence of final state interaction. Unfortunately, the $D^0\pi^0$ mode is not observed yet at this point; we expect, however, that it will be observed sometime soon as mentioned earlier.

One could go further along this line if one is bold enough. One can set $\delta = 0$ and recalculate the decay rates that would have been without the final state interaction. Then those rates may be compared with what is expected by factorization. In fact, a phenomenologically successful analysis of charm decay was performed in such manner [28]. However, there is no guarantee that all the effect of final state interaction can be taken away by this method. There may be interactions with other final states, for example.

Another possibility is to look at the azimuthal angular distribution in $B \rightarrow VV$ decays. Taking $B \rightarrow D^*\rho$ as an example, the angular distribution is given by

$$\begin{aligned} \frac{d\Gamma}{dc_D dc_\rho d\chi} &\propto (|H_+|^2 + |H_-|^2)s_D^2 s_\rho^2 + 4|H_0|^2 c_D^2 c_\rho^2 \\ &\quad + 2\text{Re}(H_+^* H_-)s_D^2 s_\rho^2 \cos 2\chi + 2\text{Im}(H_+^* H_-)s_D^2 s_\rho^2 \sin 2\chi \\ &\quad + 4\text{Re}(H_+^* H_0 + H_-^* H_0)s_D c_D s_\rho c_\rho \cos \chi + 4\text{Im}(H_+^* H_0 - H_-^* H_0)s_D c_D s_\rho c_\rho \sin \chi \end{aligned} \tag{50}$$

where $\theta_{D,\rho}$ are the polar decay angle of D and ρ decays as before, and χ is the azimuthal angle between the two decay planes. We have used a short hand: $c_D = \cos \theta_D$, $s_D = \sin \theta_D$ etc. If there is no final-state interaction and there is no CP violation, then all the helicity amplitudes are relatively real. The effect of CP violation would show up as difference of angular distribution (as well as difference in total decay rate) between B and \bar{B} decays [63]. For Cabbibo-favored modes such as $D^*\rho$, we do not expect significant CP violation. Thus, existence of terms proportional to $\sin \chi$ or $\sin 2\chi$ signals final state interaction [64]. This analysis should be able to be done with dataset presently available, but thus far not completed.

4 Suppressed Decays

Now we move to rare decays which are typically Cabbibo-suppressed. We start from charm-less two-body decays.

4.1 B Decays to Two Charmless Mesons

Each of the processes $B^0 \rightarrow K^- \pi^+$, $\pi^+ \pi^-$ could proceed through two types of diagrams: spectator and penguin (Figure 12). When there exist more than one diagram with different weak interaction phases and different final state interaction phases (i.e. strong interaction), there can be CP violating decay asymmetries [65] as seen below. Suppose two diagrams contribute to a decay $B \rightarrow f$ with amplitudes A_1 and $A_2 e^{i\delta}$ where $A_{1,2}$ are the weak amplitudes and δ is the FSI phase difference. Since only relative phases matter, the weak and strong phases of the first diagram are assumed to be zero. For the corresponding $\bar{B} \rightarrow \bar{f}$ decay, the weak phase changes its sign but the strong phase does not. This leads to a decay asymmetry:

$$\text{Amp}(B \rightarrow f) = A_1 + A_2 e^{i\delta}, \quad \text{Amp}(\bar{B} \rightarrow \bar{f}) = A_1^* + A_2^* e^{i\delta} \quad (A_1 : \text{real}) \quad (51)$$

$$(52)$$

In our case, the weak phase of each diagram is given by that of the CKM matrix elements which multiply the entire amplitude as coefficients. Thus we expect that there is a weak phase difference as shown in the figure. The strong phases, however, are difficult to estimate.

If we assume the flavor $SU(3)$ symmetry, then the ratio of amplitudes are

$$\left. \frac{K\pi}{\pi\pi} \right|_{\text{spectator}} \sim \lambda \quad \left. \frac{K\pi}{\pi\pi} \right|_{\text{penguin}} \sim \frac{1}{\lambda} \quad (53)$$

where λ is the Cabbibo suppression factor (~ 0.2). It is expected that the spectator diagram will dominate in $B^0 \rightarrow \pi^+ \pi^-$. Then if there is no penguin contribution, the $K^- \pi^+$ branching ratio should be $\lambda^2 \sim 0.04$ times smaller than that of $\pi^+ \pi^-$. Thus, if the rate of $K^- \pi^+$ is comparable or greater than $\pi^+ \pi^-$, then it is likely that the $K^- \pi^+$ rate is dominated by the penguin diagram. When there is a large disparity in magnitudes of the two diagrams, the expected CP violation will be small independent of the phases.

One should note, however, that there is a possibility that $B \rightarrow K\pi$ can occur through final state re-scatterings. This could occur through intermediate states involving two charmed mesons as

$$B^0 \rightarrow D^- D_S^+ \rightarrow K^+ \pi^-, \quad B^0 \rightarrow D^- D^+ \rightarrow \pi^- \pi^+ \quad (54)$$

Figure 12: Diagrams that can contribute to $B \rightarrow K\pi, \pi\pi$.

which corresponds to replacing the top quark loop in the penguin diagrams by a charm quark which will be on-shell as shown below and can be considered to be a dispersive version of penguin diagram.

(55)

Such process will result in a large FSI phase, and can interfere with the top quark penguin diagram to generate a CP violation as originally postulated by Bander, Silverman and Soni [66].

Approximate rate of $\pi^+\pi^-$ can be estimated from the measured $B^0 \rightarrow D^-\pi^+$ rate quite reliably:

$$Br(\pi^+\pi^-) \sim \left| \frac{V_{ub}}{V_{cb}} \right|^2 Br(D^+\pi^-) \sim 1 \times 10^{-5} \quad (56)$$

where the effect of form factor will reduce it somewhat and that of phase space will increase it somewhat. The estimation of the $K\pi$ rate requires the coefficient of the penguin operator, and the uncertainty is greater; the theoretical estimates are in the same range as the $\pi\pi$ mode [67].

Experimentally, the signature on $\Upsilon(4S)$ is a rather spectacular high-momentum back-to-back tracks of $p \sim 2.6$ GeV. This is the maximum momentum a B -decay can emit and the background is dominated by continuum events; thus, cuts are made on event shapes to reject 2-jet like events and the fast back-to-back tracks are required not to be aligned with the jet axis of event. For a $B\bar{B}$ pair event, the event shape is spherical and there is little correlation between the event axis and the direction of the back-to-back tracks. Then, as before, the energy difference ΔE and the beam-constrained mass M_B is used to select the candidates [see (3)].

When masses are correctly assigned to the tracks, the ΔE resolution is 25 MeV. The dE/dx information in the drift chamber is used to separate kaon and pion. The dE/dx resolution is 6.5% and provides 1.8σ $K - \pi$ separation per track. Each candidate is assigned the most likely masses ($\pi\pi$, $K\pi$, or KK), then ΔE is calculated. The beam-constrained mass, on the other hand, does not depend on the mass assignments and the resolution is 2.5 MeV. Figure 13(a) shows the M_B distribution for $K\pi$ and $\pi\pi$ candidates after $2\text{-}\sigma$ cut on ΔE around zero. The shaded events are the $\pi\pi$ candidates. One can see an enhancement at the nominal B mass of 5.280 GeV. The ΔE distribution after the $2\text{-}\sigma$ cut on M_B is shown in Figure 13(b). Again, there is a peak around the nominal region near $\Delta E = 0$. For the final extraction of numbers, an un-binned maximum likelihood fit

Figure 13: Sum of $K\pi$ sample and $\pi\pi$ sample. (a) The beam-constrained mass M_B after the $2\text{-}\sigma$ cut on ΔE . (b) ΔE distribution after the $2\text{-}\sigma$ cut on M_B . The shaded events are the events assigned to be $\pi\pi$.

is performed with ΔE , M_B , dE/dx , and an event shape variable as parameters. Here ΔE is calculated assuming $\pi\pi$. The result is shown in Table 7 [68].

When ΔE is calculated assuming $\pi\pi$, the value shifts down by 42 MeV if the actual tracks are $K\pi$. Since the ΔE resolution is 25 MeV, this by itself can provide 1.7σ separation between $K\pi$ and $\pi\pi$. The available particle identifications are not good enough to cleanly separate the two. When $K\pi$ and $\pi\pi$ are combined there is a substantial signal of about 3.5σ . The central value of $\pi^+\pi^-$ mode is consistent with the expected value of 1×10^5 . If we take the central value of the $K^+\pi^-$ mode at its face value, then penguin diagrams (t-loop or the re-scattering c-loop) are likely to be dominating the $K^+\pi^-$ mode.

Mode	Br(10^{-5})	Upper Limit (10^{-5})
$\pi^+\pi^-$	$1.3^{+0.8}_{-0.6} \pm 0.2$	2.9
$K^+\pi^-$	$1.1^{+0.7}_{-0.6} \pm 0.2$	2.6
K^+K^-		0.7
$K^+\pi^- + \pi^+\pi^-$	$2.4^{+0.8}_{-0.7} \pm 0.2$	

Table 7: Measured branching fractions and 90% confidence level upper limits.

4.2 b to s Radiative Decays

Another rare process a penguin diagram is expected to contribute is the radiative $b \rightarrow s$ transition through emission and re-absorption of W .

(57)

At the lowest order, the GIM suppression is operative and it depends on the top mass (m_t) strongly, and $Br(B \rightarrow X_s \gamma)$ changes from 0.5×10^{-4} at $m_t = 100$ GeV to 1.4×10^{-4} at $m_t = 200$ GeV. With QCD correction [69], the GIM suppression is loosened ('soft' GIM suppression) and as a result the rate is substantially enhanced and becomes a slow function of m_t . The enhancement factor is ~ 5 at $m_t = 120$ GeV to give $Br(B \rightarrow X_s \gamma) \sim 3.5 \times 10^{-4}$. Theoretical estimate for the exclusive mode $B \rightarrow K^* \gamma$ is more uncertain due to the unknown transition matrix element $B \rightarrow K^*$ [70]. One estimate based on HQET gives $Br(B \rightarrow K^* \gamma) = (1.4 - 4.9) \times 10^{-5}$ [71].

The experimental signature [72] is a monochromatic hard photon (2.6 GeV) recoiling against $K^* \rightarrow K \pi$ decay. We look for both $B^0 \rightarrow K^{*0} \gamma$ and $B^- \rightarrow K^{*-} \gamma$. The K^{*} 's are searched for in the modes $K^{*0} \rightarrow K^+ \pi^-$ and $K^{*-} \rightarrow K^- \pi^0, K_s^- \pi^-$. Again the background is dominated by continuum events since such high-energy photon is at the kinematic limit of B decay. The continuum backgrounds are reduced by requiring that the events be not 2-jet like and that the hard photon be not aligned to the event axis. If the photon forms a π^0 or η with another photon then it is rejected. Figure 14 shows the M_B distribution after the cut $|\Delta E| < 90$ MeV (2.2σ). There is a clear signal observed with 6.6 ± 2.8 events in B^0 mode and 4.1 ± 2.3 events in B^- modes. The branching fractions are

$$\begin{aligned} Br(B^0 \rightarrow K^{*0} \gamma) &= (4.0 \pm 1.7 \pm 0.8) \times 10^{-5}, \\ Br(B^- \rightarrow K^{*-} \gamma) &= (5.7 \pm 3.1 \pm 1.1) \times 10^{-5} \quad (\text{CLEO}) \end{aligned} \quad (58)$$

If we assume isospin symmetry, then

$$Br(B^0 \rightarrow K^{*0} \gamma) = Br(B^- \rightarrow K^{*-} \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5} \quad (59)$$

which is consistent with theoretical estimates based on the standard model where the penguin contribution dominates. Another possibility is that the $B \rightarrow K^* \gamma$ transition may occur through ψK^* by vector dominance [73]

$$B \rightarrow \psi K^* \rightarrow \gamma K^* \quad (\text{vector dominance}). \quad (60)$$

Figure 14: The M_B distribution for $B \rightarrow K^*\gamma$ after ΔE cut.

Figure 15: Single photon spectrum after continuum subtraction. The signal $b \rightarrow s\gamma$ would show up in the region 2.2 to 2.7 GeV.

or other long distance effects [74]. Such processes have been estimated and found to be at least an order of magnitude smaller than the observed rate.

The inclusive transition $B \rightarrow X_s \gamma$ can be searched by looking for the hard photon without reconstructing X_s where the mass of X_s lies in the typical strange meson region (0.5 to 2 GeV). Similar cuts as before to reduce continuum backgrounds are applied. Figure 15 shows the continuum-subtracted (see Section 1) photon spectrum. The signal region is around 2.2 to 2.7 GeV. There seems to be some enhancement, but it is not statistically significant; thus, we set an upper limit

$$Br(b \rightarrow s\gamma) < 5.4 \times 10^{-4} \quad (\text{CLEO}[75]). \quad (61)$$

Such measurement places stringent constraints on non-standard physics, in particular two-Higgs-doublet models [76]. The W-top loop can be replaced by loops involving charged Higgs, neutralinos, gluinos, and squarks etc [77]. For example, in the minimal supersymmetric model with two Higgs doublets, the mass of the CP-odd neutral Higgs A^0 (which is related to the mass of the charged Higgs) is ruled out for $m_{A^0} < 250$ GeV. [78].

Acknowledgement

The author would like to thank Mike Dugan, Giulia Ricciardi, Thorsten Ohl, and Mitch Golden for fruitful discussions. A major credit should go to the editing staff whose extraordinary patience made it possible that this article be completed. This research was supported primarily by the DOE via grant DEFG-029-1ER-40-654 and in part by NSF grant PHY-92-18167.

References

- [1] Particle Data Group, Phys. Rev. **D45** Part II (1992).
- [2] D. Bortoletto *et al.* (CLEO collaboration), Phys. Rev. **D45** (1992) 21.
- [3] H. Albrecht *et al.* (ARGUS collaboration), Z. Phys. **C48** (1990) 543.
- [4] S. Alam *et al.* (CLEO collaboration), to be published. Also, presented at the 1993 Lepton-photon conference at Cornell University.
- [5] An early attempt to predict charmed meson masses is in, A. De Rujula, H. Georgi, and S. Glashow, Phys. Rev. Lett. **37** (1976) 398. For a review, see for example, J. Gasser and H. Leutwyler, Phys. Rep **87** (1982) 78, and references therein.
- [6] J. Goity and W-S Hou, Phys. Lett. **B282** (1992) 243.
- [7] F. Abe *et al.* (CDF collaboration), FERMILAB-PUB-93-141-E (Jun 1993).
- [8] D. Buskulic *et al.* (ALEPH collaboration), Phys. Lett. **B311** (1993) 425.
- [9] P. D. Acton *et al.* (OPAL collaboration), Phys. Lett. **B295** (1992) 357.
- [10] Y. Arnoud *et al.* (DELPHI collaboration), Paper submitted to the "HEP 93" conference, Marseille, 1993.
- [11] W. Kwong and J. Rosner, Phys. Rev. **D44** (1991) 212, and references therein.
- [12] K. Han *et al.*, Phys. Rev. Lett. **55** (1985) 36; J. Lee-Franzini *et al.*, Phys. Rev. Lett. **65** (1990) 2947.
- [13] D. Akerib *et al.* (CLEO collaboration), Phys. Rev. Lett. **67** (1991) 1692.
- [14] The references quote the photon energy in the B^* rest frame. A correction of $\Delta M^2/2M_{B^*}$ is applied to convert it to the mass difference.
- [15] M. Frank and P. O'Donnell, Phys. Lett. **159B** (1985) 174; Z. Phys. **C34**, (1987) 39; K. Igi and S. Ono, Phys. Rev. **D32** (1985) 232.
- [16] E. Eichten *et al.*, Phys. Rev. **D21** (1980) 203; S. Godfrey and N. Isgur, Phys. Rev. **D32** (1985) 189.
- [17] The electromagnetic hyperfine splitting can be cleanly separated by measuring the difference of the hyperfine splittings within isospin multiplet; e.g. $[M(D^{*+}) - M(D^+)] - [M(D^{*0}) - M(D^0)]$ for D mesons: J. Amundson *et al.*, Phys. Rev. **D47**, (1993) 47.

- [18] M. Gronau, A. Nippe, and J. Rosner, Phys. Rev. **D47** (1993) 1988.
- [19] See for example, R. E. Marshak, ‘Conceptual Foundations of Modern Particle Physics’ (World Scientific, 1993); T-P Cheng and L-F Li, ‘Gauge Theory of Elementary Particle Physics’, (Oxford, 1984); T. D. Lee, ‘Particle Physics and Introduction to Field Theory’, (Harwood, 1988), to name a few.
- [20] See for example, J. Rosner, in ‘B Decays’ p312 ed. S. Stone, (World Scientific) and references therein.
- [21] A popular parametrization of the CKM matrix which is accurate to λ^3 is:

$$V \sim \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

where $\lambda \sim 0.22$ and A, ρ, η are real numbers of order 1. L. Wolfenstein Phys. Rev. Lett. **51** (1983) 1945.
- [22] F. G. Gilman and M. B. Wise, Phys. Rev. **D20** (1979) 2392.
- [23] K. G. Wilson, Phys. Rev. **179** (1969) 1499.
- [24] Originally, the calculations were done in the context of $\Delta I = 1/2$ enhancement. M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33** (1974) 108; G. Altarelli and L. Maiani, Phys. Lett. **32B** (1974) 351.
- [25] G. Altarelli, G. Curci, G. Martinelli, and S. Petrarca, Phys. Lett. **99B** (1981) 141; Nucl. Phys. **B187** (1981) 461.
- [26] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B120** (1977) 316. The purpose was to explain the $\Delta I = 1/2$ enhancement in the non-leptonic strange particle decays.
- [27] A good review is found in, M. Neubert, V. Rieckert, B. Stech, and Q.P. Xu, in ‘Heavy Flavors’ p286, ed. A. Buras (World Scientific, 1992).
- [28] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. **C34** (1987) 103.
- [29] J. Schwinger, Phys. Rev. Lett. **12** (1964) 630.
- [30] R. P. Feynman, ‘Symmetries in Elementary Particle Physics’ Ed. A. Zichichi, p111 (Academic Press, 1965).
- [31] N. Isgur and M. Wise, Phys. Lett. **B232** (1989) 113; **B237** (1990) 527.
- [32] B. Grinstein, W. Kilian, T. Mannel, and M. Wise, Nucl. Phys. **B363** (1991) 19.

- [33] J. Bjorken, Nucl. Phys. **B** (Proc. Suppl.) **11** (1989) 325.
- [34] S. Brodsky and A. Mueller, Phys. Lett. **B206** (1988) 685.
- [35] M. Dugan and B. Grinstein, Phys. Lett. **B255** (1991) 583.
- [36] A. Buras, J.-M. Gérard, and R. Rückl, Nucl. Phys. **B268** (1986) 16.
- [37] C. Reader and N. Isgur, Phys. Rev. **D47** (1993) 1007.
- [38] See for example the appendix of L. B. Okun, ‘Leptons and quarks’ (North Holland, 1982).
- [39] It is important to note that the current operator $(\bar{q}q')$ and the ‘factorized hadron operator’ $(\bar{q}q')_{\text{had}}$ are two different objects. A same current operator can contribute to more than one factorized hadron operators.
- [40] B. Blok and M. Shifman, Nucl. Phys. **B389** (1993) 534; M. Shifman, Nucl. Phys. **388** (1992) 346.
- [41] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. **C29** (1985) 637; M. Bauer and M. Wirbel, Z. Phys. **C 42** (1989) 671.
- [42] N. Isgur and M. Wise, Phys. Lett. **B237** (1990) 527.
- [43] M. Bauer and B. Stech, Phys. Lett. **B152** (1985) 380.
- [44] V. Mathur and T. Rizzo, Phys. Rev. **D16** (1977) 3343.
- [45] J. Ellis, M. Gaillard, D. Nanopoulos, Nucl. Phys. **B100** (1975) 313.
- [46] D. Fakirov and B. Stech, Nucl. Phys. **B133** (1978) 313.
- [47] H. Fritzsch and P. Minkowski, Phys. Lett. **90B** (1980) 455; M. Bander, D. Silverman, A. Soni, Phys. Rev. Lett. **44**, (1980) 7; B. Bernreuther, O. Nachtmann, and B. Stech, Z. Phys. **C4** (1980) 257. The last article also speculated a deviation from $V - A$ interaction due to soft gluon effects.
- [48] J. Rosner, Phys. Rev. **D42** (1990) 3732.
- [49] One may add a_1^2 to the expression, but what to take for ξ is not clear and the uncertainty is of order 10%. One has to be careful not to use the value of a_1 which is obtained by phenomenological fit to hadronic rates together with semileptonic form factors. That would be a biased test of factorization.
- [50] H. Albrecht *et al.* (ARGUS collaboration), Phys. Lett. **B219** (1989) 121; H. Albrecht *et al.*, Phys. Lett. **B197** (1987) 452.

- [51] D. Bortoletto *et al.*, Phys. Rev/ Lett. **63** (1989) 1667. D. Bortoletto, Ph.D. thesis, Syracuse University, 1989.
- [52] N. Isgur, D. Scora, B. Grinstein, and M. Wise, Phys. Rev. **D39** (1989) 799. B. Grinstein, M. Wise, and N. Isgur, Phys. Rev. Lett. **56** (1986) 298.
- [53] J.G. Korner and G.A. Schuler, Z. Phys. **C38** (1988) 511.
- [54] In principle, one can make a QED radiative correction; the result, however, does not vary significantly for the purpose of this analysis. See Ref [1], p VII.1.
- [55] The parameter c_V is the charge squared of the valence quark properly averaged over the quarks. For example, $c_V = 1/9$ and $4/9$ for ϕ and ψ , respectively. For ρ^0 , $c_V = |Q_u \langle u\bar{u} | \rho^0 \rangle + Q_d \langle d\bar{d} | \rho^0 \rangle|^2 = 1/2$, where Q_i is the charge of quark type i .
- [56] X.-Y. Pham and X.C. Vu, Phys. rev. **D46** (1992) 261.
- [57] D. Akerib *et al.* (CLEO collaboration), Phys. Rev. Lett. **71** (1993) 3070.
- [58] F. Butler *et al.* (CLEO collaboration), Phys. Rev. Lett. **69** (1992) 2041.
- [59] CDF internal memo. To appear in the proceedings of the Lepton-Photon Conference 1993, Cornell University.
- [60] D. Bortolletto *et al.* (CLEO collaboration), Phys. Rev. Lett. **64** (1990) 2117.
- [61] H. Albrecht *et al.* (ARGUS collaboration), Z. Phys. **C54** (1992) 1.
- [62] CLEO paper submitted to the EPS conference, Marseille 1993 (CLEO CONF 93-17).
- [63] G. Kramer and W.F. Palmer, Phys. Lett. **B279** (1992) 181.
- [64] J.G. Korner and G.R. Goldstein, Phys. Lett. **89B** (1979) 105.
- [65] A. Pais and S.B. Treiman, Phys. Rev. **D12** (1975) 218; L.B. Okun, V.I Zakharov, and B.M. Pontecorvo, Lett. Nuovo. Cim **13** (1975) 218; For a modern treatment including B decays, see for example, L. Wolfenstein, Phys. Rev. **D43** (1991) 151.
- [66] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43** (1979) 242. Also, see J.-M. Gerard and W.-S. Hou, Phys. Rev. Lett. **62** (1989) 855; D. London and R.D. Peccei, Phys. Lett. **B223** (1989) 257; I.I. Bigi, V.A. Khoze, N.G. Uraltzev, and A.I. Sanda in ‘CP Violation’, Ed. C. Jarlskog (1988) World Scientific, and references therein.
- [67] N.G. Deshpande and J. Tranpetic, Phys. Rev. **D41** (1990) 895; Phys. Rev. **D41** 2926; L.-L. Chau *et al.*, Phys. Rev. **D43** (1991) 2176.

- [68] M. Battle *et al.* (CLEO collaboration), Phys Rev. Lett **71** (1993) 3922.
- [69] B. Grinstein, R. Springer, and M.B. Wise, Phys. Lett. **B202** (1988) 138; G. Cella, G. Curci, G. Ricciardi, and A. Vicere, Phys. Lett. **B248** (1990) 181; M. Misiak, Phys. Lett. **B269** (1991) 161.
- [70] T. Altomari, Phys. Rev. **D37** (1988) 677.
- [71] A. Ali, T. Ohl, and T. Mannel, Phys. Lett. **B298** (1993) 195.
- [72] R. Ammer *et al.* (CLEO collaboration), Phys Rev. Lett. **71** (1993) 674.
- [73] N.G. Deshpande *et al.*, Phys. Lett. **B214** (1988) 467.
- [74] P. Colangelo *et al.*, Z. Phys. **C45** (1990) 575.
- [75] P. Kim, to appear in the proceedings of the heavy flavor conference, McGill University, 1993.
- [76] W.-S. Hou and R.S. Willey, Phys. Lett. **202** (1988) 591; S. Bertolini *et al.*, Nucl. Phys. **B353** (1991) 591; V. Barger *et al.*, Phys. Rev. Lett. **70** (1993) 1368.
- [77] S. Bertolini, ‘Heavy Flavors’ p202, ed. A. Buras (World Scientific,1992)
- [78] J. Hewett, Phys. Rev. Lett. **70** (1993) 1045.